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Landau-Ginzburg treatment of nuclear matter at finite temperature

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Abstract. Based on recent studies of the temperature dependence of the energy and specific heat of liquid nuclear matter, a phase transition is suggested at a temperature ~ 0.85 MeV. We apply the Landau-Ginzburg theory to this transition and determine the behaviour of the energy and specific heat close to the critical temperature in the condensed phase.

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The existence of an energy gap in the spectrum of even-even nuclei due to paired states of either protons or neutrons [1] similar to that described by Bardeen, Cooper and Schrieffer (BCS) for electrons in a superconductor [2] has led to the suggestion that nuclear matter should also exist in a condensed phase for some range of temperatures [3]. The properties of this superfluid phase in both nuclear and neutron matter have been studied in the BCS approximation using a variety of phenomenological forces [4] as well as more realistic interactions [5]. Remarkably, all calculations yield qualitatively similar results for $^1S_0$ pairing, namely that neutron matter exists in a condensed phase for k_F less than about 1.3–1.5 fm^{-1}. Recent calculations, using the Paris potential [6], by the Catania group [7] have shown that only slight deviations occur in nuclear matter. Such modifications, which can be characterized by the use of a smaller nuclear effective mass in the case of nuclear matter, are known to give rise to a slight decrease in the gap, Δ which vanishes at a temperature around 0.8 MeV at normal nuclear density. Although such calculations suggest that such a low-temperature phase should exist in both nuclear as well as neutron matter this has not been taken into account in, for example, astrophysical calculations since it is thought that it may be masked by other instabilities [8].

In field-theoretic language BCS theory is considered as the spontaneous symmetry breaking of phase symmetry. The condensed phase, *e.g.*, the superconducting phase, is characterized by an order parameter (Δ) which is zero at the critical temperature, T_c. It has been established that the order parameter and the critical temperature fulfill the following approximate relationship [9]:

$$\frac{\Delta_0}{T_\mathrm{c}} \approx 1.76\,, \tag{1}$$

where Δ_0 is the value of the energy gap at $T = 0$ and here we have taken the Boltzmann constant $k_\mathrm{B} = 1$. In the normal phase the order parameter is zero. Interestingly enough the same relationship has been found to hold in the aforementioned calculations in nuclear matter [7]. Furthermore, it has been pointed out that the same relation between Δ_0 and T_c also describes the spontaneous symmetry breaking of chiral symmetry in QCD if T_c is taken to be $2f_\pi$ [10], where f_π is the pion decay constant. In all of the aforementioned cases the order parameter is obtained from a gap-like equation with appropriate quasi-particle interactions.

Recent studies of nuclear matter have suggested that the origin of collective states may ultimately be linked to symmetry rearrangement [11]. This leads to a BCS-like condensed phase, separated from the normal phase, which has an order parameter that goes to zero at the critical temperature. Calculations in finite nuclei at finite temperature suggest that this provides a reasonable description of the vanishing of the collective degrees of freedom [12].

Recently it has been demonstrated that the low-temperature behavior of the specific heat of symmetric nuclear matter can be obtained from a finite-temperature extension of the semi-empirical mass formula [13]. The temperature dependence of the coefficients in the semi-empirical mass formula [14] was determined by fitting to the canonical ensemble average of the excitation energy of

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over 300 nuclei for temperatures $T \leq 4$ MeV, using experimental information of the energy spectra of nuclei in the mass region $22 \leq A \leq 250$. The volume term was then used to determine the temperature dependence of the energy per nucleon and specific heat of nuclear matter. This displayed some rather interesting aspects: A structure in both the energy and the specific heat was observed at temperatures between 0.5 and 1.3 MeV (the structure in the specific heat is of course more pronounced). Below this temperature the behaviour of the specific heat was quite different from that expected for a Fermi gas of free nucleons [13]. This is not unexpected as the low-lying energy spectra of most nuclei are predominantly collective in nature. Above 1.3 MeV, the specific heat was essentially linear in temperature as is the case for a Fermi gas, but with the somewhat surprising feature that the slope coefficient was considerably larger than that suggested by the Fermi gas bulk level density parameter,

$$a_v \approx \frac{1}{15}\frac{m^*}{m}, \tag{2}$$

which was obtained from a low-temperature expansion about $T = 0$ [15]. Here it was assumed that $m^* = (0.7 - 1.2)m$ [16].

In this paper we intend to investigate the aforementioned matters from the point of view of the Landau-Ginzburg theory. We propose that there is a second-order phase transition in liquid nuclear matter with a critical temperature $T_{\rm c}$ and an order parameter η. The critical temperature separates the condensed and the normal phases of the liquid. We apply Landau-Ginzburg theory to determine the thermodynamic properties of the condensed phase close to $T_{\rm c}$, from information about the normal phase. We find that the behaviour of the energy per nucleon and specific heat across the phase transition with $T_{\rm c} \sim 0.85$ MeV to be consistent with that shown in [13].

Landau and Ginzburg have provided a simple theory of phase transitions which approximates the free energy in the region around $T_{\rm c}$ and is most useful in analyzing the thermodynamics in this region. In particular, using only knowledge about the uncondensed phase one is able to make predictions about quantities in the condensed phase, such as specific heat, magnetic susceptibility and compressability. Moreover, Landau-Ginzburg theory can be derived from microscopic considerations [9].

Following the Landau-Ginzburg formulation it is necessary first to determine an expression for the free energy per nucleon $f(T)$ in both phases. In what follows the subscript 1 will refer to the lower temperature (condensed) phase, and 2 to the higher temperature (uncondensed or normal) phase. For the uncondensed phase, we take a quadratic form for the energy per nucleon which follows from a low temperature Fermi gas approximation of a normal Fermi liquid,

$$W_2(T) = a_2 + k_2T^2, \tag{3}$$

where a_2 and k_2 are constants. From the relations for the specific heat per nucleon in terms of W and the entropy per nucleon s,

$$c_V = \frac{\partial W}{\partial T} = T\,\frac{\partial s}{\partial T}, \tag{4}$$

we are able to deduce the entropy per nucleon in the uncondensed phase,

$$s_2(T) = C_2 + 2k_2T, \tag{5}$$

where C_2 is an unknown integration constant which later cancels out of the calculation. From eqs. (3) and (5) the free energy per nucleon in the higher temperature phase is given by

$$f_2(T) = a_2 - C_2T - k_2T^2. \tag{6}$$

To determine the free energy per nucleon in the condensed phase, we make use of the Landau expansion [17] for the free energy per nucleon in terms of an order parameter η, which goes to zero at the transition to the uncondensed phase. This order parameter is related to the presence of pairing expected at lower temperatures and vanishes with the pairing gap Δ at some critical temperature $T_{\rm c}$. The free energy per nucleon expansion to order η^4 is

$$f_1(T,\eta) = f_2 + A\eta^2 + B\eta^4. \tag{7}$$

Here A and B are functions of temperature and we have assumed that the states with $\eta = 0$ and $\eta \neq 0$ are of different symmetry. In this case it can be shown the linear term in η must be set equal to zero. Furthermore if the critical point is also a stable point, *e.g.*, if f_1 as a function of η is a minimum at $\eta = 0$, then the third-order term in η should be zero and at the critical point [17]

$$A = 0\,, \qquad B > 0\,.$$

The order parameter is determined by requiring the condensed phase to be stable below $T_{\rm c}$ (*i.e.* f_1 should be minimized w.r.t. η). This leads to

$$f_1 = f_2 - \frac{A^2}{4B}. \tag{8}$$

Further, since A is of opposite sign in the condensed and uncondensed phases, whilst B is strictly positive [17], the lowest-order expansion of A in $T - T_{\rm c}$ can be parametrized as

$$A(T) = a(T - T_{\rm c})\,2\sqrt{B(T_{\rm c})}. \tag{9}$$

Note especially that $a > 0$ is an essential requirement following from the phase diagram [17]. Substituting for $A(T)$, the free energy per nucleon near $T_{\rm c}$ is given by

$$f_1(T) = (a_2 - a^2T_{\rm c}^2) + (2a^2T_{\rm c} - C_2)T - (k_2 + a^2)T^2, \tag{10}$$

where f_2 is taken from eq. (6).

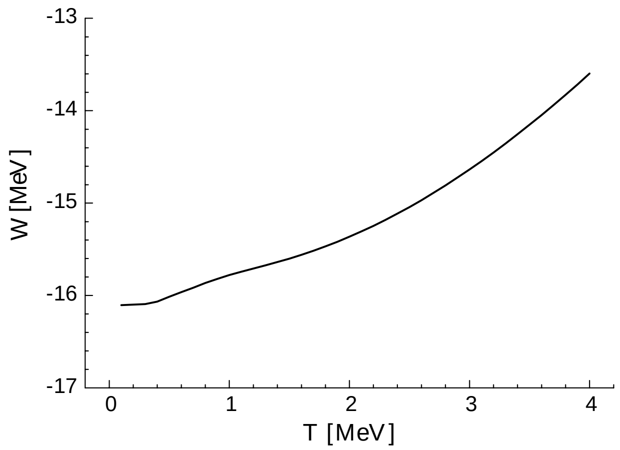

Fig. 1. The energy per nucleon of symmetric nuclear matter *vs.* T obtained ref. [13].

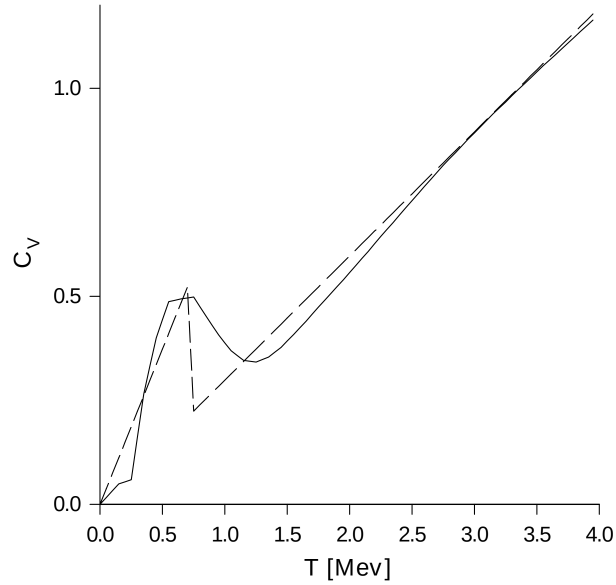

Fig. 2. The specific heat per nucleon of symmetric nuclear matter *vs.* T. The solid curve has been obtained from ref. [13]. The dashed curve is obtained from Landau-Ginzburg theory with $k_2 = \frac{1}{6.7}$ MeV^{-1} and $T_c = 0.85$ MeV. For illustrative purposes the condensed phase specific heat asymptote ($T \to T_c$) is shown for $k_1 = 2k_2$.

From the free energy per nucleon given by eq. (10), we can now determine the energy per nucleon in the condensed phase near T_c,

$$W_1(T) = (a_2 - a^2T_c^2) + (a^2 + k_2)T^2 \quad (11)$$
$$= a_1 + k_1T^2. \quad (12)$$

Comparing this to the uncondensed phase (eq. (3)) we note that the T dependence is also quadratic, but has a larger coefficient. Thus the specific heat is discontinuous across the phase transition, and is necessarily larger ($k_1 > k_2$) in the condensed phase.

We now compare the structure of $W_1(T)$ and $W_2(T)$ to what has been determined from the finite temperature extension of the semi-emperical mass formula [13]. Before proceeding, it should be noted that the energy per nucleon in nuclear matter is obtained from the volume term of the binding energy for finite sized nuclei whose inividual partition functions are analytic. It may be anticipated that any sharp features (*e.g.*, kink in the energy per nucleon and discontinuity in specific heat Δc_V) will appear smoothed out. Thus whilst comparison is still possible at a qualitative level, it is difficult to obtain quantitative estimates for the critical temperature and the discontinuity in the specific heat.

In the region (0.50–1.3 MeV) the energy per nucleon from [13] is observed to show a peak above the simple T^2 behavior (see fig. 1). This is in good agreement with what might be expected from a smoothed out downwards kink in W at T_c, which follows from eqs. (3) and (12). Furthermore, the specific heat (see fig. 2) in [13] shows a sharp drop in the region (0.5–1.3 MeV) which agrees well with a smoothed out discontinuous drop ($= 2(k_1 - k_2)T_c$). It should be noted that the specific heat above 1.3 MeV is very nearly linear, supporting the use of a quadratic temperature dependence of W_2, and that the slope below 0.5 MeV is greater than that above 1.3 MeV, which is in good agreement with our prediction that $k_1 > k_2$. In fig. 1, k_2 is determined from the fit in [13] to the uncondensed phase parameters for the energy per nucleon given in eq. (3), of $k_2 = 1/6.7$ MeV^{-1} and $a_2 = -16$ MeV. In addition, we took $T_c \sim 0.85$ MeV and for purely illustrative purposes have shown a possible asymptote line ($T \to T_c$) for the specific heat in the condensed phase when $k_1 = 2k_2$.

If we treat the uncondensed phase as a Fermi gas of quasi-particles with a Landau mass $m_L \sim m$, based on the linear behaviour of the specific heat, we can estimate the jump in specific heat at the transition to a condensed phase, where there is pairing with an associated energy gap Δ. This is given by [9],

$$\Delta c_V \approx 1.43 c_V, \quad (13)$$

where c_V is the specific heat per nucleon in the uncondensed phase. Using $k_2 = 1/6.7$ MeV^{-1} and assuming $T_c \sim 0.85$ MeV we find $\Delta c_V \sim 0.36$, which is in remarkably good agreement with the behaviour of the specifc heat per nucleon shown in fig. 2.

At temperatures considerably higher than T_c, the energy per particle given by eq. (3) will become positive. It is reasonable to identify this with a transition from a Fermi liquid to a Fermi gas, at temperature T_{LG}. Using eq. (3) with the fitted parameters a_2 and k_2 taken from [13], we estimate this transition temperature to be at $T_{LG} \approx 10$ MeV. This compares favourably with $T_{LG} \approx$ 15–20 MeV in [18] and $T_{LG} \approx 5$ MeV (finite nuclei) [19].

It is established that there are three phases of the nuclear matter: the condensed liquid phase, the normal liquid phase and the gas phase. The thermodynamic properties of the condensed phase near T_c are determined from the properties of the normal phase by using the Landau-

Ginzburg theory. The transition from condensed to normal liquid is a second-order phase transition while the liquid to gas is a first-order phase transition. A study of the nuclear matter at zero temperature must include the ramifications of a condensed phase. The normal phase is present only between the two critical temperatures. Clearly the energy per nucleon obtained in [13] at temperatures above 1.3 MeV is much stiffer than that of a Fermi gas of free nucleons, which is often used in many astrophysical calculations [8] which in turn should affect neutrino production rates in stars. As this is the major cooling mechanism in these objects it would be interesting to see precisely how important this deviation is.

Lack of experimental data on nuclear matter at finite temperature makes further refinement of the model difficult. Experimental determination of the thermodynamic properties in heavy-ion collisions would be extremely helpful for understanding the properties of nuclear matter at finite temperature.

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